1 Ideas and questions

1.1 Generalize regularity and E-inversivity with one definition (10.2003)

Let S be a semigroup, $E(S) = \{x \in S \mid x^2 = x\}$ the set of idempotents and $a \in S$.

Element a is called ***regular***, if there exists $x \in S$ such that $a = axa$. S is called regular, if all its elements are regular.

Element a is called E-***inversive***, if there exists $x \in S$ such that $ax \in E(S)$. S is called E-inversive, if all its elements are E-inversive.

It is possible to introduce the concept of k-inversive element and a respective semigroup. Regular semigroup is 1-inversive, E-inversive semigroup is 2-inversive, and the rest is shown in the follwing table:

$$\begin{array}{rlccl}
1-inversive & \forall a \in S\ \exists x \in S: & a & = & axa \\
2-inversive & \forall a \in S\ \exists x \in S: & ax & = & axax \\
3-inversive & \forall a \in S\ \exists x \in S: & axa & = & axaxa \\
4-inversive & \forall a \in S\ \exists x \in S: & axax & = & axaxax \\
5-inversive & \forall a \in S\ \exists x \in S: & axaxa & = & axaxaxa \\
\dots & \dots & \dots & \dots & \dots \\
2k-inversive & \forall a \in S\ \exists x \in S: & (ax)^k & = & (ax)^{k+1} \\
(2k+1)-inversive & \forall a \in S\ \exists x \in S: & (ax)^k a & = & (ax)^{k+1} a \\
\dots & \dots & \dots & \dots & \dots
\end{array}$$

1.2 Certain subsets of a semigroup (10.2003)

In Applied Mathematics E-Notes **1**(2001), 111-117, there was a very interesting article by Shigeru Kobayashi and Haruki Shimada called *$\mathcal{L}$-Classes of Inverse Semigroups*. There were many interesting results, but one of them was that an $\mathcal{L}$-class L_a of an inverse semigroup is a subsemigroup iff for every $b \in L_a$, $b^{-1}b = b^{-2}b^2$. And only in one place in some proof it was written out that this actually means $b\mathcal{L}b^2$. The same goes dually for $\mathcal{R}$-classes, too, of course.

But what does it mean? An $\mathcal{L}$-class turns out to have a property, that for being a subsemigroup it is not necessary to take $b, c \in L_a$ and check for $bc \in L_a$, but instead we may use $c = b$ and check for $bb \in L_a$. So an $\mathcal{L}$-class is a subsemigroup iff it is already closed under taking squares (instead of all products).

Now we can define a new class of subsets T of a semigroup S which have the property

$$(\forall x \in T:\ x^2 \in T) \Rightarrow (\forall x, y \in T:\ xy \in T).$$

If we take the meet semilattice of these subsets (in an inverse semigroup) with respect to $\cap$, then what role have $\mathcal{L}$-classes in that semilattice?

There's a lot of classes of semigroups related to regular and inverse semigroups — how about them if we ask when their $\mathcal{L}$-class is a subsemigroup?

1.3 Simple question? (10.2003)

If we have $x = x^k = x^l$, $k > l > 1$, $k, l \in \mathbb{N}$, in a semigroup S, what follows from these equalities? $x = x^{m(k,l)}$, but what is $m(k, l)$ equal to? The process for calculating $m(k, l)$ looks something like modified Euler's algorithm for gcd...

1.4 $\frac{1}{3}-$ and $\frac{2}{3}-$associative dense groupoids (15.01.2004)

Let G be a groupoid (a nonempty set with a binary operation (multiplication)) and let $G^* = G \setminus \{0, 1\}$ if G contains a 0, 1 or both of them, otherwise $G^* = G$.

Definition 1.4.1 Let's call G $\frac{1}{3}-$***associative dense***, if

$$\forall a \in G\, \exists b, c \in G^* \ : \ a(bc) = (ab)c.$$

Definition 1.4.2 Let's call G $\frac{2}{3}-$***associative dense***, if

$$\forall a, b \in G\, \exists c \in G^* \ : \ a(bc) = (ab)c.$$

I didn't bother myself to find better names for those groupoids, I used what seemed most natural to me. The idea came into my head when I read that E-inversive semigroups (see 1.1) have also been called E-dense (sometimes or somewhere).

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